Lambda-Terms Λ

- $\mathcal{C} \subseteq \Lambda$
- $\mathcal{V} \subseteq \Lambda$
- $(t_1\, t_2) \in \Lambda$, if $t_1, t_2 \in \Lambda$
- $\lambda x.t \in \Lambda$, if $x \in \mathcal{V}$ and $t \in \Lambda$

Substitutions

- $x[x/t] = t$
- $y[x/t] = y$ for all $y \in \mathcal{V}$ with $y \neq x$
- $c[x/t] = c$ for all $c \in \mathcal{C}$
- $(r_1\, r_2)[x/t] = (r_1[x/t]\; r_2[x/t])$ for all $r_1, r_2 \in \Lambda$
- $(\lambda x.r)[x/t] = \lambda x.r$
- $(\lambda y.r)[x/t] = \lambda y.(r[x/t])$, if $y \neq x$ and $y \notin \mathrm{free}(t)$
- $(\lambda y.r)[x/t] = \lambda y'.(r[y/y'][x/t])$, if $y \neq x$, $y \in \mathrm{free}(t)$, $y' \notin \mathrm{free}(r) \cup \mathrm{free}(t)$

α-Reduction

- $\lambda x.t \ \rightarrow_\alpha \ \lambda y.t[x/y]$, if $y \notin \mathrm{free}(t)$
- $t_1 \ \rightarrow_\alpha \ t_2$ implies $(t_1\, r) \ \rightarrow_\alpha (t_2\, r)$, $(r\, t_1) \ \rightarrow_\alpha (r\, t_2)$, $\lambda y.t_1 \ \rightarrow_\alpha \ \lambda y.t_2$

β-Reduction

- $(\lambda x.t)\, r \ \rightarrow_\beta \ t[x/r]$
- $t_1 \ \rightarrow_\beta \ t_2$ implies $(t_1\, r) \ \rightarrow_\beta (t_2\, r)$, $(r\, t_1) \ \rightarrow_\beta (r\, t_2)$, $\lambda y.t_1 \ \rightarrow_\beta \ \lambda y.t_2$

Example

- $(\lambda x.x)\, \texttt{Zero} \ \rightarrow_\beta \ \texttt{Zero}$
- $(\lambda xy.x\, y)\, y \ \rightarrow_\alpha \ (\lambda xy'.x\, y')\, y \ \rightarrow_\beta \ \lambda y'.y\, y'$
- $(\lambda xy.x\, y)\, y \ \rightarrow_\beta \ \lambda y'.y\, y'$