Recursive Path Order
Let \( \sqsubseteq \) be well-founded order on \( \Sigma \) (precedence). We have \( s \succ_{rpo} t \) iff

- \( s = f(s_1, \ldots, s_n) \) and \( s_i \succ_{rpo} t \) for an \( i \) or
- \( s = f(s_1, \ldots, s_n), t = g(t_1, \ldots, t_m), f \sqsubseteq g, \) and \( s \succ_{rpo} t_j \) for all \( j \) or
- \( s = f(s_1, \ldots, s_n), t = f(t_1, \ldots, t_n), \{s_1, \ldots, s_n\} \succ_{rpo} \{t_1, \ldots, t_n\} \)

RPO with Status \( \succ_{rpos} \)
Assign permutation of 1, \ldots, \( n \) or “multiset” to every \( n \)-ary symbol \( f \), compare arguments lexicographically in this order or as multiset.

\[
\begin{align*}
\text{sum}(\emptyset, y) &\rightarrow y \\
\text{sum}(\text{succ}(x), y) &\rightarrow \text{sum}(x, \text{succ}(y)) \\
\text{plus}(\emptyset, y) &\rightarrow y \\
\text{plus}(\text{succ}(x), y) &\rightarrow \text{succ}(\text{plus}(y, x))
\end{align*}
\]
• $s$ and $t$ are **unifiable** iff there exists a **unifier** $\sigma$ with $s\sigma = t\sigma$

• **Unification Problem** $S = \{ s_1 =? t_1, \ldots, s_n =? t_n \}$

• $\sigma \in U(S)$ iff $s_i\sigma = t_i\sigma$ for all $1 \leq i \leq n$

• $\sigma$ is more general than $\sigma'$ iff there is a substitution $\delta$ with $\sigma' = \sigma\delta$

**Example** $S = \{ g(f(x), y) =? g(y, f(z)) \}$

$U(S)$ contains

$\sigma = \{ x/z, y/f(z) \}$

$\sigma_1 = \{ x/a, y/f(a), z/a \}$

$\sigma_2 = \{ x/f(z), y/f(f(z)), z/f(z) \}$

$\sigma_3 = \{ z/x, y/f(x) \}$ etc.

$\sigma$ and $\sigma_3$ are **most general** unifiers, since

$\sigma_1 = \sigma\delta_1$ with $\delta_1 = \{ z/a \}$

$\sigma_2 = \sigma\delta_2$ with $\delta_2 = \{ z/f(z) \}$

$\sigma_3 = \sigma\delta_3$ with $\delta_3 = \{ z/x \}$