Algorithm UNIFY($S$)

1. While there exists an $S'$ with $S \implies S'$, let $S := S'$ and goto 1.
2. If $S$ is in solved form, return $\sigma_S$. Otherwise, return "False".
Delete  \( S \uplus \{ t = ? t \} \) \( \rightarrow \) \( S \)

Reduce Term  \( S \uplus \{ f(s_1, \ldots, s_n) = ? f(t_1, \ldots, t_n) \} \) \( \rightarrow \) \( S \uplus \{ s_1 = ? t_1, \ldots, s_n = ? t_n \} \)

Exchange  \( S \uplus \{ t = ? x \} \) \( \rightarrow \) \( S \uplus \{ x = ? t \} \), if \( t \not\in V \)

Reduce Var.  \( S \uplus \{ x = ? t \} \) \( \rightarrow \) \( \{ u\sigma = ? v\sigma \mid u = ? v \in S \} \uplus \{ x = ? t \} \)

, if \( \sigma = \{ x/t \}, x \not\in V(t), x \in V(S) \)

Algorithm **UNIFY(\( S \))**

1. While there exists an \( S' \) with \( S \rightarrow S' \), let \( S := S' \) and goto 1.
2. If \( S \) is in solved form, return \( \sigma_S \). Otherwise, return “False”.

**Thm. 5.1.9** (Soundness of the Unification Algorithm)

(a) The relation \( \rightarrow \) is well founded.
(b) If \( S \rightarrow S' \), then we have \( U(S) = U(S') \).
(c) If \( S \) is solvable and in normal form w.r.t. \( \rightarrow \), then \( S \) is in solved form.
(d) The algorithm UNIFY terminates and is correct.