$$
\begin{aligned}
\mathsf{f}(x, \mathsf{f}(y, z)) &\to \mathsf{f}(\mathsf{f}(x, y), z) && (G1)\\
\mathsf{f}(x, \mathsf{e}) &\to x && (G2)\\
\mathsf{f}(x, \mathsf{i}(x)) &\to \mathsf{e} && (G3)
\end{aligned}
$$

Critical Pairs:

$$
\begin{tikzcd}
 & \mathsf{f}(x, \mathsf{f}(y, \mathsf{e})) \arrow[dl, "(G1)"'] \arrow[dr, "(G2)"] & \\
\mathsf{f}(\mathsf{f}(x, y), \mathsf{e}) \arrow[d, "(G2)"'] & & \mathsf{f}(x, y) \\
\mathsf{f}(x, y) & &
\end{tikzcd}
\qquad
\begin{tikzcd}
 & \mathsf{f}(x, \mathsf{f}(y, \mathsf{i}(y))) \arrow[dl, "(G1)"'] \arrow[dr, "(G3)"] & \\
\mathsf{f}(\mathsf{f}(x, y), \mathsf{i}(y)) & & \mathsf{f}(x, \mathsf{e}) \arrow[d, "(G2)"] \\
 & & x
\end{tikzcd}
$$

$$
\begin{tikzcd}
 & \mathsf{f}(x', \mathsf{f}(x, \mathsf{f}(y, z))) \arrow[dl, "(G1)"'] \arrow[dr, "(G1)"] & \\
\mathsf{f}(\mathsf{f}(x', x), \mathsf{f}(y, z)) \arrow[d, "(G1)"'] & & \mathsf{f}(x', \mathsf{f}(\mathsf{f}(x, y), z)) \arrow[d, "*"', "(G1)"] \\
\mathsf{f}(\mathsf{f}(\mathsf{f}(x', x), y), z) & & \mathsf{f}(\mathsf{f}(\mathsf{f}(x', x), y), z)
\end{tikzcd}
$$

Algorithm CONFLUENCE($\mathcal{R}$)

Input: A terminating TRS $\mathcal{R}$.
Output: "*True*", if $\mathcal{R}$ is confluent and "*False*", otherwise

1. Compute all critical pairs $CP(\mathcal{R})$ of $\mathcal{R}$.
2. If $CP(\mathcal{R}) = \varnothing$, then return "*True*" and stop.
3. Choose $\langle s, t \rangle \in CP(\mathcal{R})$.
4. Reduce s and t as long as possible.
   In this way, one obtains the normal forms s' and t'.
5. If $s' \neq t'$ then return "*False*" and stop.
6. Let $CP(\mathcal{R}) = CP(\mathcal{R}) \setminus \{\langle s, t \rangle\}$.
7. Go back to Step 2.